CO1

# K.S. SCHOOL OF ENGINEERING AND MANAGEMENT, BANGALORE - 560109

## DEPARTMENT OF MECHANICAL ENGINEERING

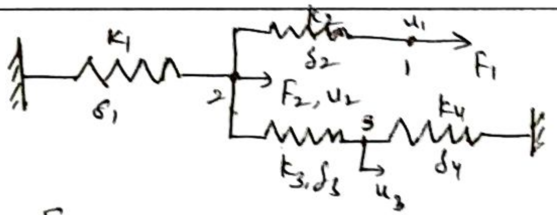
## I SESSIONAL TEST SCHEME & SOLUTION 2022 – 23 EVEN SEMESTER

**SET-A**

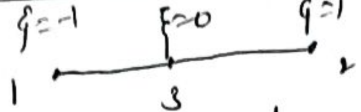
| USN | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| **Degree** | : | **B. E** | **Semester** | : | **VI** |
| **Branch** | : | **Mechanical Engineering** | **Date** | : | **19-04-2023** |
| **Course Title** | : | **Finite Element Method** | **Course Code** | : | **18ME61** |
| **Duration** | : | **90 Minutes** | **Max Marks** | : | **30** |

**Note: Answer ONE full question from each part**

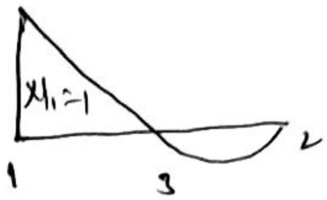
| Q. No. | Questions with Scheme & Solution | Marks |
|---|---|---|
| | **PART-A** | |
| 1(a) Sol | **Steps in FEM**<br>1. Discretization or subdivision of the domain<br>2. Selection of the displacement functions<br>3. Formulation of the elemental stiffness matrix<br>4. Assembly of stiffness matrix<br>5. Applying the boundary conditions<br>6. Determining the nodal displacement<br>7. Determining the elemental stress and strain<br>Explain | ① 2m<br>② & ③ 2m<br>④ to ⑦ 1m. |
| (b) Sol | <br>$\Pi = \frac{1}{2}\left[k\delta_1^2 + k_2\delta_2^2 + k_3\delta_3^2 + k_4\delta_4^4\right] - \left[F_1u_1 + F_2u_2 + F_3u_3\right]$<br>$\delta_1 = u_2, \quad \delta_2 = u_1 - u_2, \quad \delta_3 = u_3 - u_2 \quad \delta_4 = -u_4$<br>$\therefore \Pi = \frac{1}{2}\left[k_1u_1^2 + k_2(u_1 - u_2)^2 + k_3(u_3 - u_2)^2 + k_4u_3^2\right] - \left[F_1u_1 + F_2u_2 + F_3u_3\right]$ — ①<br>$\frac{\partial \Pi}{\partial u_1} = 0, \quad \frac{\partial \Pi}{\partial u_2} = 0, \quad \frac{\partial \Pi}{\partial u_3} = 0.$ — ②<br>Solve all the equations,<br>$u_1 = 2.86m$<br>$u_2 = 1.81m$<br>$u_3 = 1.05m$ — ③ | eqv-① 2m<br>eqn② = 2m<br>eqn③ 1m |

**(c) Sol**



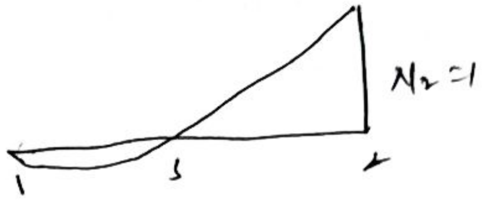

Consider a bar element in natural coordinate system.

$$\boxed{N_1 = \frac{\xi(\xi-1)}{2}}$$



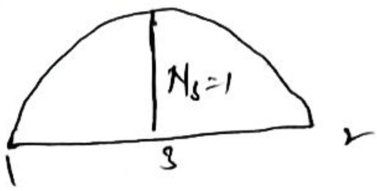

2m

$$\boxed{N_2 = \frac{\xi(\xi+1)}{2}}$$




2m

$$\boxed{N_3 = 1-\xi^2}$$

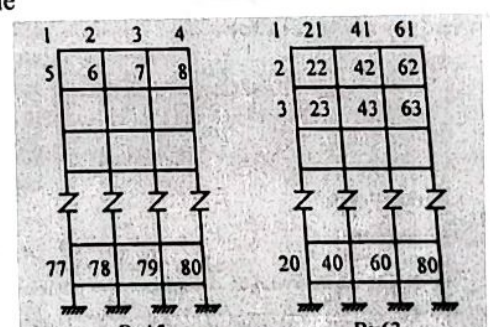



1m

**OR**

**2(a) Sol**

Node numbering scheme




B=15 (a) along the shorter dimension

B=63 (b) along the longer dimension

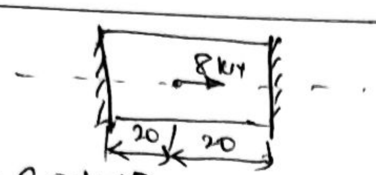
2m

Since most of the matrices involved in the finite element analysis are symmetric and banded. The required computer storage can be considerably reduced by storing only the elements involved in half bandwidth instead of storing the whole matrix. The bandwidth of the assemblage matrix depends on the node numbering scheme and the number of degrees of freedom considered per node. If we can minimize the bandwidth, the storage requirements, as well as solution time can also be minimized

3m

| | | |
|---|---|---|
| **(b) Sol** | see below | eqn ① = 2m; eqn ② = 2m; eqn ③ = 1m |
| **(c) Sol** | see below | eqn ④ = 2m; eqn ① = 2m; eqn ③ = 1m |

**(b) Sol**




$\pi = SE + WP$  — eqn ① = 2m

$$\pi = \frac{EA}{2}\int_{2}\left(\frac{\partial u}{\partial x}\right)^2 dx - Pu_1$$ — eqn ② = 2m

Displacement function

$U = a_0 + a_1x + a_2x^2$ — eqn ③ = 1m

Applying BC's and simplify

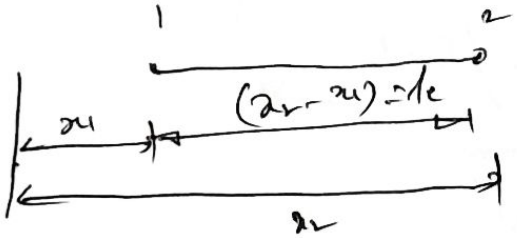
$$\pi = \frac{EA}{2}\int_0^{40}\left(40a_2^2x^2 + 1600a_2^2 - 160a_2^2x\right)dx + 8\times10^{3}\times400a_2 \quad —①$$

$\frac{\partial\pi}{\partial a_2} = 0$, $a_2 = -0.0214\times10^{-3}$ —②

$\therefore \boxed{u_1 = 0.00857\,mm}$ —③

at $x = 20$, $\sigma = 0$,
$x = 0$, $\sigma = 60\,N/mm^2$
$x = 40$, $\sigma = -60\,N/mm^2$ —④

**(c) Sol**




eqn ④ = 2m

$U_1$ & $U_2$ = Displacement at node 1 & Node 2

$N_1$ & $N_2$ = Shape functions at node 1 & Node 2 — eqn ① = 2m

Assume polynomial function:-

$U = a_0 + a_1x$ —① — eqn ③ = 1m

After simplification

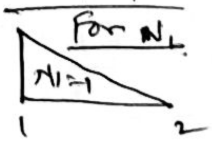
$$\left.\begin{aligned} a_1 &= \frac{U_2 - U_1}{x_2 - x_1} \\ a_0 &= \frac{U_1x_2 - U_2x_1}{x_2 - x_1}\end{aligned}\right\} —②$$

$$U = \frac{(x_2 - x)}{(x_2 - x_1)}u_1 + \frac{(x - x_1)}{(x_2 - x_1)}u_2$$

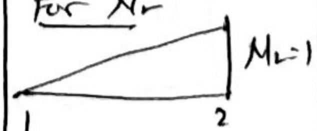
$$N_1 = \left(\frac{x_2 - x}{x_2 - x_1}\right), \quad N_2 = \left(\frac{x - x_1}{x_2 - x_1}\right) —④$$

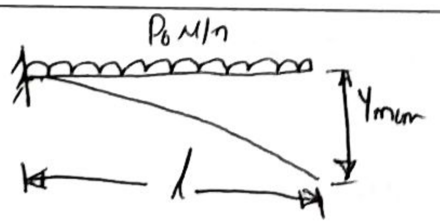
Variations:-

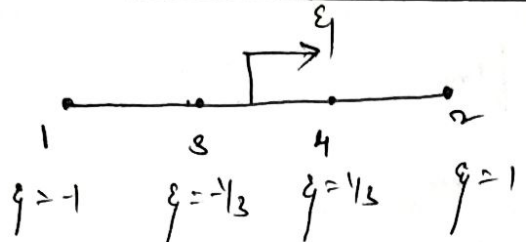
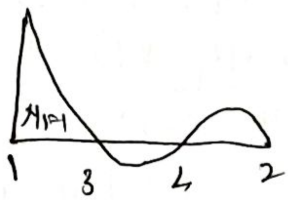
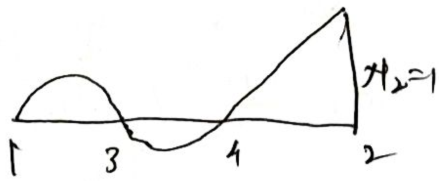
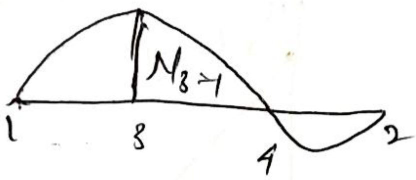
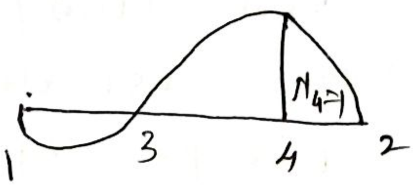
For $N_1$



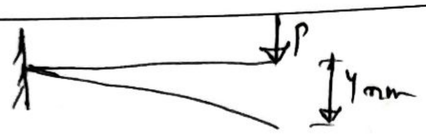

For $N_2$



<table>
<tr><td></td><td>PART-B</td><td></td></tr>
<tr><td>3(a)<br>Sol</td><td>The finite element is a numerical technique used to obtain the approximate solution as the element size is reduced. This sequence of approximate solution is still converge to the exact solution, if the interpolation model satisfy the following conditions called convergence<br>1. The displacement field must be continuous<br>2. The displacement must be compatible between adjacent element<br>3. The displacement field must represent constant strain rates of the element<br>4. The displacement function must represent the rigid body displacentent of the element<br>5. The displacement function must have the no of generalized co-ordinates equal to the nodal displacement or degree of freedom of the element<br>6. Spatial isotropy<br>Explain</td><td>Each point carries 1 m</td></tr>
<tr><td>(b)<br>Sol</td><td><br><br>$\Pi = SE + WP$<br>$\Pi = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx - \int_0^l P_0\, y\, dx \rightarrow (0)$<br><u>Assume displacement function:-</u><br>$y = a_0 + a_1x + a_2x^2 + a_3x^3$<br>Applying B.C.<br>$y = a_2x^2 + a_3x^3$<br>$\therefore \Pi = \frac{EI}{2}\left[4a_2^2L + 12a_3^2L^3 + 12a_2a_3L^3\right] - P_0\left[a_2\frac{L^3}{3} + a_3\frac{L^4}{4}\right] \rightarrow (1)$<br>$\frac{\partial\Pi}{\partial a_2} = 0, \quad \frac{\partial\Pi}{\partial a_3} = 0 \quad — (2)$<br>$\boxed{a_2 = \frac{5PL^3}{24EI}}$ $\boxed{a_3 = \frac{-PL}{12EI}}$ — (3)<br>$\therefore \boxed{y_{max} = \frac{PL^4}{8EI}}$</td><td>equ (0) = 2 m<br><br>2 equ (2) = 2 m<br><br>equ (3) = 1 m</td></tr>
</table>

<table>
<tr><td>(c)<br>Sol</td><td>
<br>Consider a cubic bar element in natural Coordinate system.</td><td>1m</td></tr>
<tr><td></td><td>$N_1 = -\frac{9}{16}\left(\xi^2 - \frac{1}{9}\right)(\xi - 1)$ 
</td><td>1m</td></tr>
<tr><td></td><td>$N_2 = \frac{9}{16}\left(\xi^2 - \frac{1}{9}\right)(\xi + 1)$ 
</td><td>1m</td></tr>
<tr><td></td><td>$N_3 = \frac{27}{16}\left(\xi^2 - 1\right)\left(\xi - \frac{1}{3}\right)$ 
</td><td>1m</td></tr>
<tr><td></td><td>$N_4 = -\frac{27}{16}\left(\xi^2 - 1\right)\left(\xi + \frac{1}{3}\right)$ 
</td><td>1m</td></tr>
<tr><td></td><td>OR</td><td></td></tr>
<tr><td>4(a)<br>Sol</td><td>Variational Principles in FEM<br>1. Principle of virtual work<br>A body is in equilibrium, when the internal virtual work equal to the external virtual work<br>2. Principle of virtual displacement<br>Are those satisfy the boundary conditions and ensures that no discontinuities such as voids<br>3. Potential energy<br>It is defined as Sum strain energy due to internal stresses and work potential due to external force<br>4. Minimum Potential energy Principle<br>It states that partial differentiation of potential energy with respect to variables is always equal to zero<br>Explain</td><td>1m<br>1m<br>1m<br>1m</td></tr>
</table>

**(b) Sol**




$\pi = \int (U + W_P)$

$\pi = \frac{EI}{2}\int \left(\frac{d^2y}{dx^2}\right)^2 dx - P y_{max}$ — eqn ① = 2m

Assume displacement function — eqn ② = 2m

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

Apply B.C's & simplify

$y = a_2 x^2 + a_3 x^3$

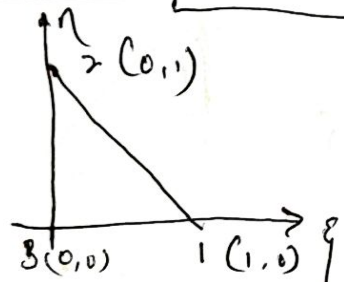
$\therefore \pi = EI\left[2a_2^2 L + 6a_3^2 L^3 + 6a_2 a_3 L^2\right] - P\left[a_2 L^2 + a_3 L^3\right]$ — ① — eqn ③ = 1m

$\frac{\partial \pi}{\partial a_2} = 0$ $\quad \frac{\partial \pi}{\partial a_3} = 0$

$\boxed{a_2 = \frac{PL}{2EI}}$ $\quad \boxed{a_3 = -\frac{P}{6EI}}$ — ②

$\therefore \boxed{y_{max} = \frac{PL^3}{3EI}}$ — ③

**(c) Sol**



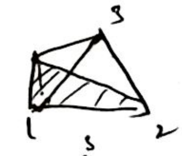

Consider a CST element in natural co-ordinate system — 2m

to derive the shape function the basic equation is

$N = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta$

Shape function at node 1

$\boxed{N_1 = \xi}$ — 1m




Shape function at node 2

$\boxed{N_2 = \eta}$ — 1m


Shape function at node 3

$\boxed{N_3 = 1 - \xi - \eta}$ — 1m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DEPARTMENT OF MECHANICAL ENGINEERING

I SESSIONAL TEST SCHEME & SOLUTION 2022 – 23 EVEN SEMESTER

SET-B

| USN | | | | | | | | | | |
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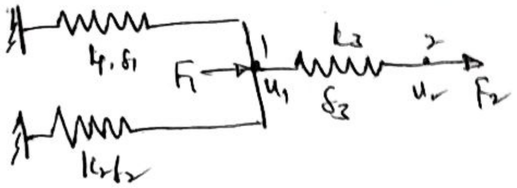
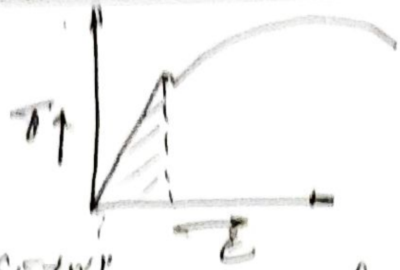
| | | | | | |
|---|---|---|---|---|---|
| **Degree** | : **B. E** | | **Semester** | : | **VI** |
| **Branch** | : **Mechanical Engineering** | | **Date** | : | **19-04-2023** |
| **Course Title** | : **Finite Element Method** | | **Course Code** | : | **18ME61** |
| **Duration** | : **90 Minutes** | | **Max Marks** | : | **30** |

**Note: Answer ONE full question from each part**

<table>
<tr><th>Q. No.</th><th>Questions with Scheme & Solution</th><th>Marks</th></tr>
<tr><td></td><td>PART-A</td><td></td></tr>
<tr><td>1(a)<br>Sol</td><td>Discretization Process<br>Dividing the continuum into number of elements is called Discretization, This is equivalent to replacing the domain having an infinite number of degrees of freedom by a system having finite number of degrees of freedom the basic requirements during Discretization are<br>a) Type of elements<br>b) Size of elements<br>c) Location of nodes<br>d) Number of element<br>e) Node numbering scheme<br>f) Automatic mesh generation<br>Explain</td><td>1m<br>a,b,c – 2m<br>d,e,f – 2m</td></tr>
<tr><td>(b)<br>Sol</td><td><br>$\pi = SE - WP$<br>$\pi = \frac{1}{2}\left[k_1u_1^2 + k_2u_1^2 + k_3(u_2-u_1)^2\right] - \left[F_1u_1 + F_2u_2\right]$ — (1)<br>$\frac{\partial\pi}{\partial u_1} = 0 \quad \frac{\partial\pi}{\partial u_2} = 0$ — (2)<br>$\boxed{u_1 = 1.1m \quad u_2 = 1.725m}$ — (3)</td><td>eq(1) = 2m<br>eq(2) = 2m<br>eq(3) = 1m</td></tr>
</table>

**PART-B**

**3(a) Sol**




$\pi = SE + WP$ — 1m

Strain energy can be found by the area under the curve

$\therefore SE = \frac{1}{2}\int \sigma^T \epsilon \, dv$ — ① — eq (1) = 2m

$WP = WP_{b} + WP_{T} + WP_{\text{point load}}$ — ②

$WP = \int_v u^T f \, dv + \int_s u^T T \, ds + \sum u_i^T P_i$ — eq ② = 2m

$\pi = \frac{1}{2}\int_v \sigma^T \epsilon \, dv - \left[\int_v u^T f \, dv + \int_s u^T T \, ds + \sum u_i^T P_i\right]$ — ③

**(b) Sol**


1m

$\pi = SE + WP$

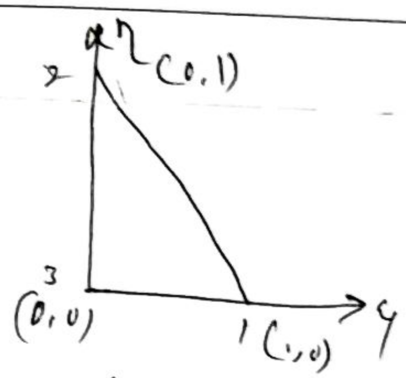
$\pi = \frac{EI}{2}\left[4a_2^2 L + 12 a_3^2 L^3 + 12 a_2 a_3 L^2\right] - P_0\left[\frac{a_2 L^3}{3} + \frac{a_3 L^4}{4}\right]$ — ① — 1m

$\frac{\partial \pi}{\partial a_2} = 0, \quad \frac{\partial \pi}{\partial a_3} = 0$ — ② — 1m

$\boxed{a_2 = \frac{5PL^3}{24EI}}$

$\boxed{a_3 = -\frac{PL}{12EI}}$ — ③ — 1m

$\boxed{y_{max} = \frac{PL^4}{8EI}}$ — ④ — 1m



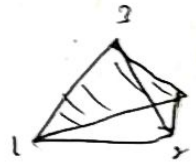

1m

(c) Sol

Shape function at node 1


$\boxed{N_1 = \xi}$ 1m

Shape function at Node 2



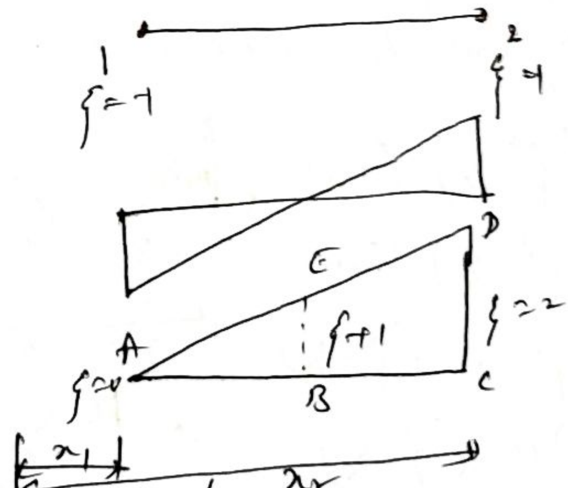

$\boxed{N_2 = \eta}$ 1m

Shape function at node 3


$\boxed{N_3 = (1-\xi-\eta)}$ 2m

**OR**

4(a) Sol



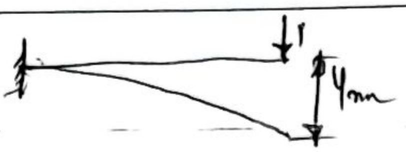

2m

from similar Δle 2m

$$\frac{AB}{AC} = \frac{BE}{CD}$$

$$\boxed{\xi = \frac{2(x-x_1)}{(x_2-x_1)} - 1}$$

**(b) Sol**



$\pi = SE + WP$

$$\pi = \frac{EI}{2}\int_0^L \left(\frac{d^2y}{dx^2}\right)^2 dx - P\,y_{max}$$

Assuming displacement function

$$y = a_0 + a_1x + a_2x^2 + a_3x^3$$

Applying B.C's

$$y = a_2x^2 + a_3x^3$$

$$\therefore \pi = EI\left[2a_2^2L + 6a_3^2L^3 + 6a_2a_3L^2\right] - P\left[a_2L^2 + a_3L^3\right] \quad -①$$

$$\frac{\partial \pi}{\partial a_2} = 0, \quad \frac{\partial \pi}{\partial a_3} = 0 \quad -②$$

$$\boxed{a_2 = \frac{PL}{2EI}} \qquad \boxed{a_3 = -\frac{P}{6EI}}$$

$$\therefore \boxed{y_{max} = \frac{PL^3}{3EI}} \quad -③$$

equ① = 2m

equ② = 2m

equ③ = 1m

**(c) Sol**

$\xi = -1 \quad \xi = -1/3 \quad \xi = 1/3 \quad \xi = 1$

1m

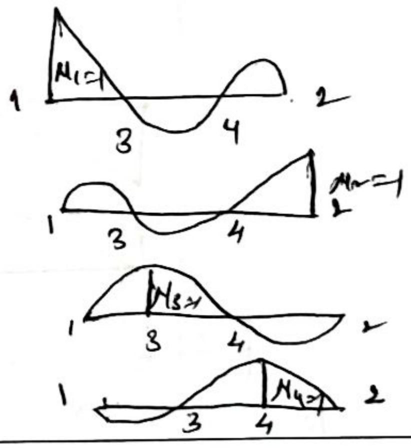
Consider a ~~bar~~ cubic bar element in natural coordinate system

$$N_1 = -\frac{9}{16}\left(\xi^2 - 1/9\right)(\xi - 1)$$ 1m

$$N_2 = \frac{9}{16}\left(\xi^2 - 1/9\right)(\xi + 1)$$ 1m

$$N_3 = \frac{27}{16}\left(\xi^2 - 1\right)(\xi - 1/3)$$ 1m

$$N_4 = -\frac{27}{16}\left(\xi^2 - 1\right)(\xi + 1/3)$$ 1m



Course In charge | Head - Dept | Principal