K.S. SCHOOL OF ENGINEERING AND MANAGEMENT, BANGALORE - 560109

DEPARTMENT OF MECHANICAL ENGINEERING

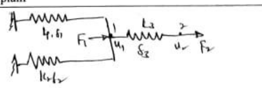
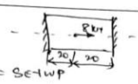
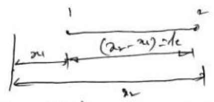
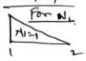
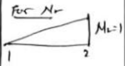
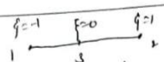
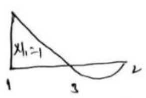
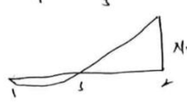
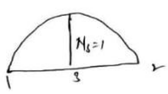
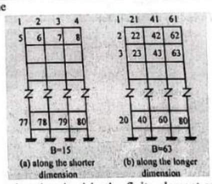
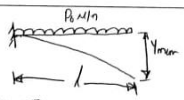
I SESSIONAL TEST SCHEME & SOLUTION 2022 – 23 EVEN SEMESTER

SET-B

USN

Degree	: B. E	**Semester**	: VI
Branch	: Mechanical Engineering	**Date**	: 19-04-2023
Course Title	: Finite Element Method	**Course Code**	: 18ME61
Duration	: 90 Minutes	**Max Marks**	: 30

Note: Answer ONE full question from each part

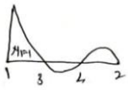
Q. No.	Questions with Scheme & Solution	Marks
	PART-A	
1(a) Sol	Discretization Process Dividing the continuum into number of elements is called Discretization, This is equivalent to replacing the domain having an infinite number of degrees of freedom by a system having finite number of degrees of freedom the basic requirements during Discretization are a) Type of elements b) Size of elements c) Location of nodes d) Number of element e) Node numbering scheme f) Automatic mesh generation Explain	1m a,b,c – 2m d,e,f – 2m
(b) Sol	 $\pi = SE + WP$ $\pi = \frac{1}{2}[k_1u_1^2 + k_2u_1^2 + k_3(u_2-u_1)^2] - [F_1u_1 + F_2u_2]$ — ① $\frac{\partial\pi}{\partial u_1} = 0 \quad \frac{\partial\pi}{\partial u_2} = 0$ — ② $\boxed{u_1 = 1.1m \quad u_2 = 1.725m}$ — ③	eq① = 2m eq② = 2m eq③ = 1m
(b) Sol	 $\pi = SE + WP$ $\pi = \frac{EA}{2}\int\left(\frac{\partial u}{\partial x}\right)^2 dx - Pu_1$ Displacement function $u = a_0 + a_1x + a_2x^2$ Applying BC's and Simplify $\pi = \frac{EA}{2}\int_0^{40}(16a_2^2x^2 + 1600a_2^2 - 160a_2^2x)dx + 8\times10^3 \times 400a_2$ — ① $\frac{\partial\pi}{\partial a_2} = 0, \quad a_2 = -0.0214\times10^{-3}$ — ② $\therefore \boxed{u_1 = 0.00857mm}$ — ③ at $x = 20$, $\sigma = 0$ $x = 0$, $\sigma = 60N/mm^2$ $x = 40$, $\sigma = -60N/mm^2$ — ④	eq① = 2m eq② = 2m eq③ = 1m
(c) Sol	 u_1 & u_2 = Displacement at node 1 & Node 2 N_1 & N_2 = Shape function at node 1 & Node 2 Assume polynomial function:- $u = a_0 + a_1x$ — ① After Simplification $a_1 = \frac{u_2 - u_1}{x_2 - x_1}$ $a_0 = \frac{u_1x_2 - u_2x_1}{x_2 - x_1}$ — ② $u = \frac{(x_2 - x)}{(x_2 - x_1)}u_1 + \frac{(x - x_1)}{(x_2 - x_1)}u_2$ $N_1 = \left(\frac{x_2 - x}{x_2 - x_1}\right), \quad N_2 = \left(\frac{x - x_1}{x_2 - x_1}\right)$ ④ Variation!- For N_1  For N_2 	eq① = 2m eq② = 2m eq③ = 1m
(c) Sol	 Consider a bar element in natural Coordinate system. $\boxed{N_1 = \frac{\xi(\xi - 1)}{2}}$  $\boxed{N_2 = \frac{\xi(\xi + 1)}{2}}$  $\boxed{N_3 = 1 - \xi^2}$ 	2m 2m 1m
	OR	
2(a) Sol	Node numbering scheme  Since most of the matrices involved in the finite element analysis are symmetric and banded. The required computer storage can be considerably reduced by storing only the elements involved in half bandwidth instead of storing the whole matrix. The bandwidth of the assemblage matrix depends on the node numbering scheme and the number of degrees of freedom considered per node. If we can minimize the bandwidth, the storage requirements, as well as solution time can also be minimized	2m 3m
	PART-B	
3(a) Sol	The finite element is a numerical technique used to obtain the approximate solution as the element size is reduced. This sequence of approximate solution is still converge to the exact solution, if the interpolation model satisfy the following conditions called convergence 1. The displacement field must be continuous 2. The displacement must be compatible between adjacent element 3. The displacement field must represent constant strain rates of the element 4. The displacement function must represent the rigid body displacentent of the element 5. The displacement function must have the no of generalized co-ordinates equal to the nodal displacement or degree of freedom of the element 6. Spatial isotropy Explain	Each point carries 1m
(b) Sol	 $\pi = SE + WP$ $\pi = \frac{EI}{2}\int_0^l\left(\frac{d^2y}{dx^2}\right)dx - \int_0^l P_0 y\,dx$ — ① Assume displacement function:- $y = a_0 + a_1x + a_2x^2 + a_3x^3$ Applying BC's. $y = a_2x^2 + a_3x^3$. $\therefore \pi = \frac{EI}{2}\left[4a_2^2L + 12a_3^2L^3 + 12a_2a_3L^2\right] - P_0\left[a_2\frac{L^3}{3} + a_3\frac{L^4}{4}\right]$ — ① $\frac{\partial\pi}{\partial a_2} = 0, \quad \frac{\partial\pi}{\partial a_3} = 0.$ — ② $\boxed{a_2 = \frac{5PL^2}{24EI}} \quad \boxed{a_3 = \frac{-PL}{12EI}}$ — ③ $\therefore \boxed{y_{max} = \frac{PL^4}{8EI}}$	eq① = eq② = eq③ =

(c) Sol

$\xi=-1 \quad \xi=-1/3 \quad \xi=1/3 \quad \xi=1$

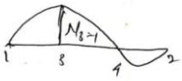
Consider a cubic bar element in natural Coordinate system. — 1m

$N_1 = -\frac{9}{16}\left(\xi^2 - 1/9\right)(\xi - 1)$ — 1m

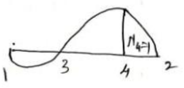


$N_2 = \frac{9}{16}\left(\xi^2 - \frac{1}{9}\right)(\xi + 1)$ — 1m

$N_3 = \frac{27}{16}\left(\xi^2 - 1\right)\left(\xi - 1/3\right)$ — 1m



$N_4 = -\frac{27}{16}\left(\xi^2 - 1\right)\left(\xi + 1/3\right)$ — 1m

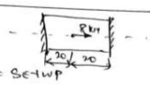


OR

4(a) Sol

Variational Principles in FEM 1. Principle of virtual work	
A body is in equilibrium, when the internal virtual work equal to the external virtual work	1m
2. Principle of virtual displacement	
Are those satisfy the boundary conditions and ensures that no discontinuities such as voids	1m
3. Potential energy	
It is defined as Sum strain energy due to internal stresses and work potential due to external force	1m
4. Minimum Potential energy Principle	
It states that partial differentiation of potential energy with respect to variables is always equal to zero Explain	1m

(b) Sol



$\pi = SE + WP$ — eq(1) = 1m

$\pi = \frac{EA}{2}\int\left(\frac{\partial u}{\partial x}\right)^2 dx - Pu_1$ — eq(2) = 2m

<u>Displacement function</u>

$u = a_0 + a_1x + a_2x^2$ — eq(3) = 1m

Applying BC's and simplify

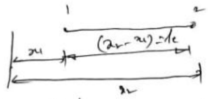
$\pi = \frac{EA}{2}\int_0^{40}\left(4a_2^2x^2 + 1600a_2^2 - 160a_2^2x\right)dx + 8\times10^3 \times 400a_2$ — (1)

$\frac{\partial\pi}{\partial a_2} = 0, \quad a_2 = -0.0214\times10^{-3}$ — (2)

$\therefore \boxed{u_1 = 0.00857\,mm}$ — (3)

at $x=20, \quad \sigma = 0$
$x=0, \quad \sigma = 60\,N/mm^2$
$x=40, \quad \sigma = -60\,N/mm^2$ — (4)

(c) Sol



eq(1) = 2m

u_1 & u_2 = Displacements at node 1 & Node 2

N_1 & N_2 = Shape functions at node 1 & Node 2 — eq(2) = 2m

Assume polynomial function:-

$u = a_0 + a_1x$ — (1) — eq(3) = 1m

After Simplification

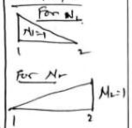
$a_1 = \frac{u_2 - u_1}{x_2 - x_1}$

$a_0 = \frac{u_1x_2 - u_2x_1}{x_2 - x_1}$ — (2)

$u = \frac{(x_2 - x)}{(x_2 - x_1)}u_1 + \frac{(x - x_1)}{(x_2 - x_1)}u_2$

$N_1 = \left(\frac{x_2 - x}{x_2 - x_1}\right), \quad N_2 = \left(\frac{x - x_1}{x_2 - x_1}\right)$ — (4)

<u>Variations!-</u>



Sample CIE Scheme & Solution